

# FINITE DIFFERENCE TIME DOMAIN (FDTD) MODELING OF GROUND PENETRATING RADAR (GPR) FOR HETEROGENEOUS MEDIA BY USING AN EFFICIENT ROBUST ALGORITHM

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## Abstract

*Ground Penetrating Radar (GPR)* is a non-invasive tool commonly used to characterize the physical properties of the subsurface. The translation of the physical measurements to geologic and hydrogeologic conditions is the culmination of many geophysical investigations. Numerical modeling increases the applicability of GPR in the geophysics area when applied parallel to the GPR data, allowing understanding of the effects of complex electromagnetic phenomena by defining and solving problems, as well as predicting the performance of radar in a complex heterogeneous environment. Finite difference time domain (FDTD) has been widely used for numerical modeling of GPR, but most of the previous algorithms are limited in their ability to model electrical conductivity and permittivity. In this research, a highly efficient robust algorithm was developed to enhance the effectiveness of the FDTD forward modeling in surroundings characterized by an arbitrary distribution of all electrical properties in three dimensional space. The modeling algorithm was developed for a heterogeneous half-space medium to facilitate statistical modeling of complex distributions of hydrologic properties in the subsurface. The results produced by the simulation reveal high accuracy using the robust algorithm to optimize three dimensional FDTD forward modeling of GPR responses in heterogeneous surroundings.

## INTRODUCTION

Ground Penetrating Radar (GPR) has been widely applied to probe the near subsurface of the earth in hydrogeologic studies (Benson and Yuhr, 1995; Beres and Haeni, 1991; Davis and Annan, 1989; Peters *et al.*, 1994; Van Overmeeren, 1994), and more recently (Daniels, 1996). GPR equipment, data processing, display, and interpretation techniques have improved over the past decade to the point whereby GPR is now routinely used for many environmental and engineering applications (Pyke *et al.*, 2006).

The level of the water table is vitally important to the stability of highways. An elevated water table often caused by heavy rain can cause subsidence on the highways and may result in serious injury or death. There have been numerous examples in the literature of GPR measurements illustrating reflections from the water table. It is clear that in some geologic conditions (e.g. clean sand, gravel, or limestone) a strong GPR reflection is frequently recorded from the water table. However, interpreters of GPR data often find that water table reflections are absent when clayey soils are present. A fundamental question that needs answering is: Where is the reflection occurring? Is the reflected signal coming from the water table (zone of 100% saturation), the top of the capillary fringe, or the top of the funicular zone? This is not a critical question if one needs to establish an approximate depth to the water table for regional hydrologic studies. However, the source of the reflection is important for determining soil type and soil moisture in the shallow subsurface.

The goal of this research was to improve the understanding of the electromagnetic wave response to the water table in the subsurface using GPR and then to apply forward modeling to help predict the behavior. To provide high speed and accuracy in forward modeling, a computer program was developed to process all necessary inputs for forward modeling. Details and advantages of this program will be explained in the next sections.

## FORWARD MODELING

Forward modeling of GPR data in conductive dielectric media is essential to understand the complex electromagnetic phenomena and to define and to solve the inverse problem in order to clarify the subsurface dielectric properties. The theoretical basis for GPR wave propagation is found in Maxwell's equations. Electromagnetic wave simulation methods with application to GPR include ray theory techniques (Cai and McMechan, 1995), pseudispectral methods (Zeng *et al.*, 1995), integral equation methods (Michalski and Mosig, 1997), and Finite Difference Time Domain (FDTD) solutions of Maxwell's equations (Greenfield and Wu, 1991).

The FDTD method is a robust computational technique useful for modeling electromagnetic processes. It has been used in induction electromagnetic (EM) (Hohmann, 1988; Wang and Hohmann, 1993) and in GPR (Roberts, 1994; Bergmann *et al.*, 1999) studies. Both EM and GPR follow a method first described by Yee (1966). Yee's method uses a staggered-grid approximation of Maxwell's curl relations (Holt, 2004):

$$\nabla \times \mathbf{e} = -\mu \frac{\partial \mathbf{h}}{\partial t} \quad (1),$$

$$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\epsilon \partial \mathbf{e}}{\partial t} \quad (2),$$

The staggered grid (Figure 1) is represented by a perfect square. The electric field ( $\mathbf{E}$ ) values are obtained from the center point of each of the 12 edges. Similarly, a sample is taken from the center of each of the 6 faces, providing the magnetic field ( $\mathbf{H}$ ) values. The electric and magnetic field vectors are offset in time by one-half of a time step, therefore updating either the electric or magnetic fields requires only previously calculated values (Holt, 2004; Eyuboglu, 2005).

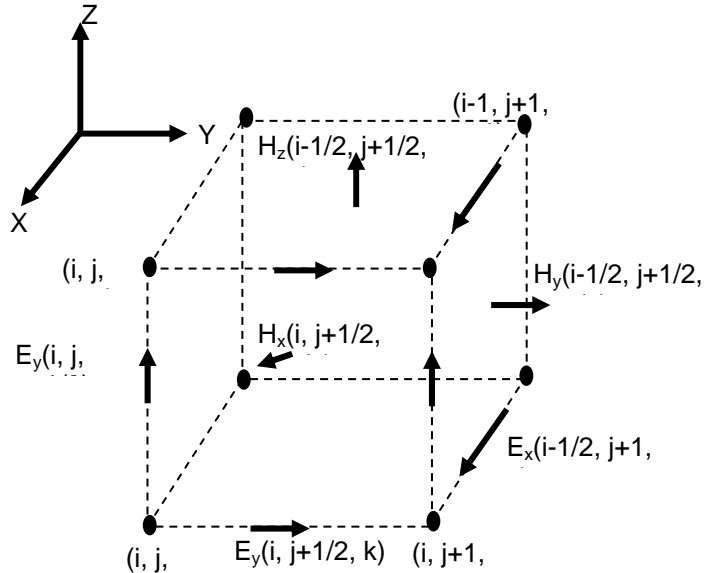


Figure 1. Staggered-grid where electric and magnetic field vectors are located tangent to the edges and normal to the sides, respectively (modified from Yee, 1966; Holt, 2004).

The FDTD modeling used in this research was created by the Electro Science Laboratory at The Ohio State University (Rama Rao *et al.*, 2001). The modeling program was developed using the C++ programming language. The anisotropic perfectly matched layer (PML) makes this method unique and different from the other modeling programs. PML was used to prevent reflections from the truncated boundary. The conductivity of the PML is chosen to establish a quadratic profile in the outward direction along which the attenuation is preferred (Rama Rao *et al.*, 2001). The PML layers are 20 cells thick on all

sides. The permittivity and conductivity of the PML layer are matched to the adjacent Maxwell region for impedance continuity. Additionally for this forward modeling, in an effort to improve the process of user input and provide a “friendlier” interface, a complimentary Interactive Data Language (IDL)-based code was developed to build the model that is used in the FDTD code. This new program utilizes graphical user interfaces (GUIs) and provides an explanation of parameters needed for the model, has the ability to assign electrical and magnetic properties to individual cells within the model (Figure 2), assigns the cell dimensions automatically based upon maximum and minimum frequencies of transmitter as well as dielectric properties of the layers. This program gives the opportunity to the user to assign cell dimensions manually. Additionally, the number of lines and traces, intervals between the traces, and location of the transmitter and receiver can be controlled with this program. This robust technique provides a tremendous time advantage, enhances the control of the user and makes the FDTD code more visual and easily understood.

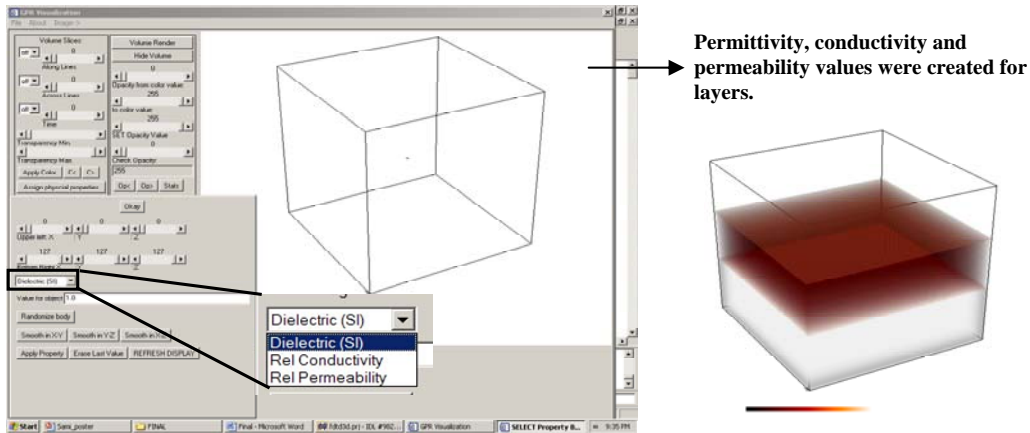


Figure 2. Left panel shows the preparation process for layers in our computer program and right panel shows the layers after the process.

To determine the accuracy of the FDTD model, the model result was compared with the real GPR result. The FDTD model was utilized in a forward model capacity to determine the response of GPR signal to subsurface conditions such as those occurring in a physical model of a GPR outdoor experiment. A high pass frequency of 5 MHz and a low pass frequency of 1100 MHz were selected for the test case to establish a 500 MHz central frequency as used in outdoor GPR tank experiment (Pyke *et al.*, 2006). The computational grid increment in the test case was 0.012 m in both vertical and horizontal axes, the time step in the modeling test case was 0.023 ns. The transmitter and receiver were 4 cells apart and were located in the center of the top of the first layer for the test case. The incident wave of FDTD modeling was excited by a second derivative of the gaussian wave as a dipole in the Maxwell region;

$$J_y(t) = 2 / T_b * (2 * X^2 - 1) * \exp(-X * X) \quad (3),$$

where

$$X = (t - t_0) / T_b \quad (4),$$

$$T_0 = 3.2 T_b \quad (5),$$

$$T_b = \sqrt{\frac{2 \log\left(\frac{f_{\max}}{f_{\min}}\right)}{\pi(f_{\max}^2 - f_{\min}^2)}} \quad (6).$$

This pulse was chosen for being the most similar to a GPR incident wave signal during an outdoor tank experiment (Pyke *et al.*, 2006).

Modeling was performed using the same physical and electrical properties as the outdoor tank experiment (Figure 3). Subsurface conditions for each test case were based on volume estimates and layer interfaces.

The modeled incident wave for the test case has higher amplitude than GPR incident waves measured in the outdoor tank experiment. This higher amplitude is a result of the dipole assumption for the model. The excitation was generated in the model as a dipole in the Maxwell region but the antenna in the tank experiment excitation was not an ideal dipole. For this reason, the ratio of the amplitude of reflection signal to incident wave for the experiment is different than that seen in the modeling results, as shown in Figure 3. However, the arrival times of the reflections are accurate and the amplitudes can be compared in a relative sense. A high similarity is observed between real and FDTD results as seen in Figure 3.

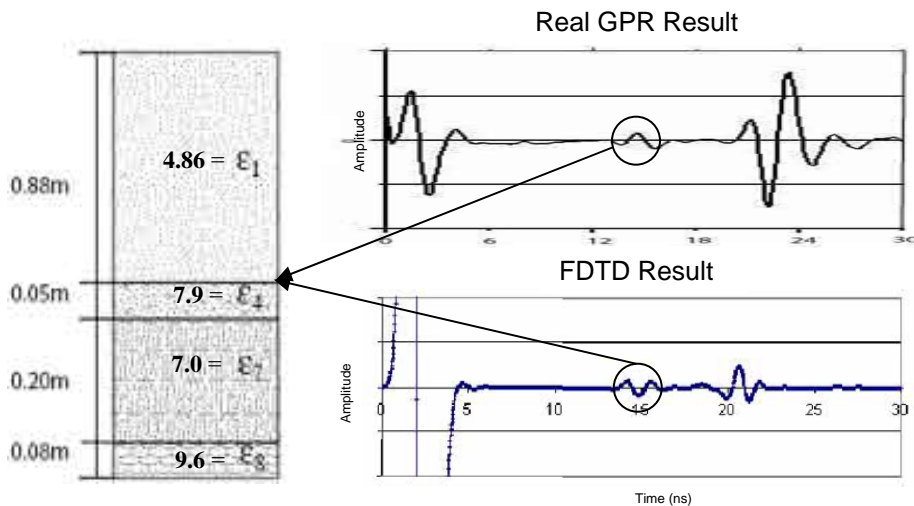


Figure 3. Left panel shows the schematic diagram of the real experiment. On right panel, the top graph represents the real GPR result, and the bottom graph represents the FDTD result.

Despite the near identical results between real GPR and FDTD, the model does not represent the real condition due to the homogenous environment in the model. Complex geology at a field site is often simplified and assumes: geological formations are homogenous in order to reduce computational time, as algorithms for heterogeneous modeling are cumbersome and slow (Yeh, 1998). However, with the physical properties view, the variable saturation zones above the water table are intermediary and based upon the soil type and the lateral heterogeneity of the materials. In the vertical direction, the borders between all of these zones are dependent upon the types of soil, regolith, or rock the area presents; the current and historical climatic conditions; and the regional and the local geomorphology of the site. These same factors affect the heterogeneity of the vadose zone in the horizontal (lateral) direction and generally compound the problems of defining the different regions of moisture in the vadose zone (Daniels *et al.*, 2005).

In order to emulate a realistic vadose zone as explained above, we wanted to create a heterogeneous media for FDTD modeling. To create a heterogeneous environment, arbitrary distribution of all electrical properties were established by using the stochastic process. The stochastic process (random field or random function) is a collection of an infinite number of random variables in space or time (Yeh, 2002). Our process relies on the stochastic representation of heterogeneity of geological media.

The original code of heterogeneity was developed by Dr. T-C. Jim Yeh (Department of Hydrology and Water Resources, The University of Arizona) using Fortran computer program language. That language in the original code was changed and adapted to our IDL code to create a heterogeneous medium for the FDTD modeling. The geophysical properties (dielectric constant, conductivity and permeability) of the media can be automatically randomized during the input preparation process in IDL. In an attempt to control the heterogeneity in the FDTD code, a simple two-layer model was developed. A dielectric constant of 30.0 and an electric conductivity of 0.001 S/m was used for the first layer and a dielectric constant of 80.0 and an electric conductivity of 0.001 S/m was used for the second layer. A schematic diagram of the model, view of the IDL code during the randomization and histogram graph of the dielectric constant value 30 with a variance of 0.002 are shown in Figure 4.

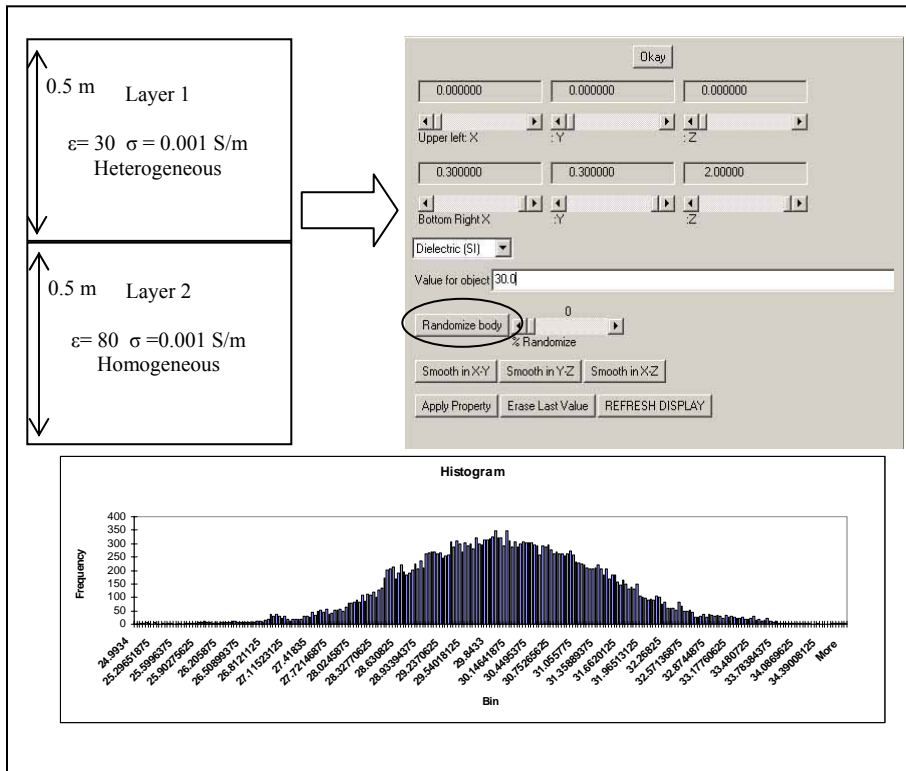


Figure 4. Top left panel shows the schematic diagram of the model and top right panel shows the view from IDL program during the process of randomization for the first layer with dielectric constant 30 and the variance 0.002. The bottom panel shows the histogram graph of the randomized data (dielectric constant value of 30).

The FDTD model was run using a heterogeneous and a homogenous model prepared in our IDL program. A high pass frequency of 100 MHz and a low pass frequency of 600 MHz were selected for both test cases to establish a 300 MHz central frequency. The computational grid increment in all test cases was 0.0091 m in both vertical and horizontal axes, the time steps in all modeling test cases were 0.0175 ns. The transmitter and receiver were 4 cells apart and were located in the center of the top of the first layer. The FDTD results for both models are shown in Figure 5. The calculated travel time to reach the second layer is 18.26 ns and the travel time from the FDTD result is 18.19 ns. The effect of the heterogeneity is clearly visible (right panel) in Figure 5.

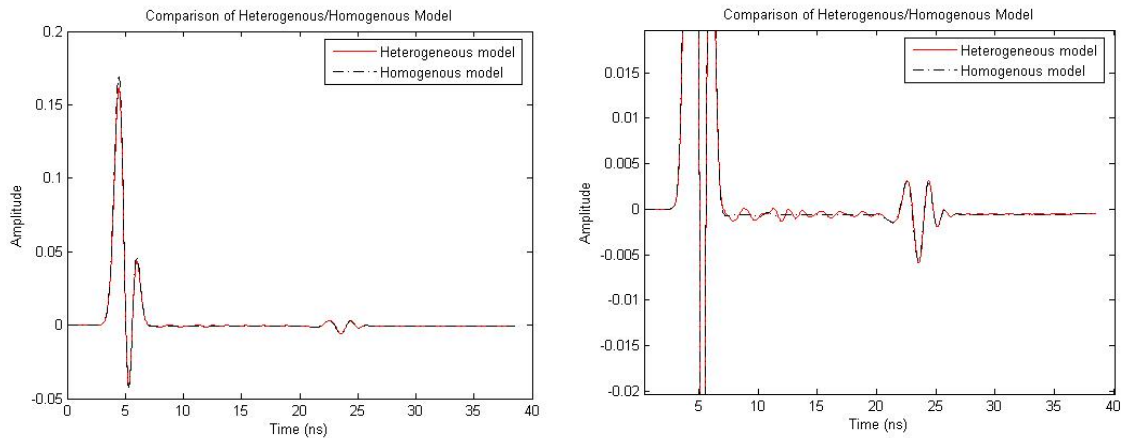


Figure 5. Left panel is the FDTD result for both homogenous and heterogeneous model. Right panel is the zoom-in view of the FDTD results.

Even though the effect of heterogeneity has been well demonstrated in the model using a one dimensional graph, a 3-D pictorial-type representation provides better understanding of the system as a whole and better demonstrates the effect of the heterogeneity. By using “Gphyz” program developed by Dr. Jeff Daniels, (School of Earth Science at The Ohio State University ([www.geology.ohio-state.edu/~jeff](http://www.geology.ohio-state.edu/~jeff))) all GPR lines collected and plot them in a 3-D fashion. Figure 6 is an example of the heterogeneous model with 3-D representation combining all GPR data from the modeling results.

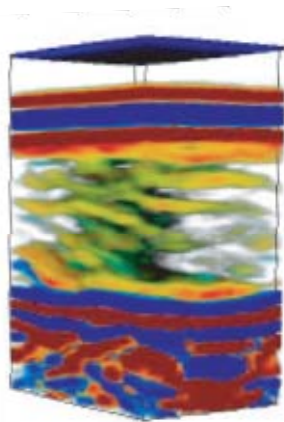


Figure 6. View of 3-D GPR model with heterogeneity using “Gphyz” program

## CONCLUSIONS

FDTD modeling has historically been applied using the assumption of homogeneity. Despite the knowledge that this practice creates a superficial condition for modeling, it involves fewer measurements, less time and less expense. Our work was aimed at creating a usable program to produce heterogeneous layers by using randomization then be applied in FDTD modeling. The randomization option has added exceptionality to our program. Real and FDTD results show high accuracy between each other and the effect of the heterogeneity was well demonstrated. By taking modeling to next level, our program recommends itself for use with inverse modeling to characterize hydrologic properties in the subsurface.

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